M-H Aero Report AD 5143-TR9

STUDY OF AUTOMATIC CONTROL SYSTEMS FOR HELICOPTERS

Analytical Representation of Reciprocating Engine Dynamics

Contract No. Nonr-929(00)
Phase II

Curtis Brown
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31 August 1954

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Aeronautical Controls

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FOREWORD

This technical report was prepared by the Research Department, Aeronautical Division, Minneapolis-Honeywell Regulator Company, under Navy Contract No. Nonr-929(00), administered by the Office of Naval Research. The contract was initiated under the research project identified by Expenditure Accounts 4600 (Research Navy) and 46832 (Aircraft and Facilities Navy). This is a contract for research involving the study of helicopter control systems from the point of view of automatic control of flight attitude, altitude, and rotor rpm.

Minneapolis-Honeywell is pleased to acknowledge the splendid cooperation displayed by representatives of the Wright Aeronautical Corporation in providing steady-state performance data, inertia data, and information on operating characteristics of the Wright R-1300 and R-1820 reciprocating engines.

ABSTRACT

In conjunction with automatic control system studies being conducted under Navy Contract Nonr 929(00), especially as pertaining to helicopter rotor rpm control, it was deemed essential to obtain an accurate representation of helicopter reciprocating engine dynamics. This report discusses the procedure followed in deriving a suitable transfer function for the engine from a study of steady-state performance data and from analysis of available engine transient characteristic data. It is believed that the transfer function thus obtained is on firmer engineering ground than previously available representations of reciprocating engine dynamics. Recommendations are made for additional investigation.

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SECTION I

INTRODUCTION

The research study being conducted under Navy Contract Nonr 929 (00) has as one of its primary considerations the simulation of helicopter and engine to the highest practicable degree of accuracy consistent with time allotments. The dynamics of the research vehicle (the Sikorsky HRS-3) were determined from basic concepts of aerodynamics and mechanics to the required degree of accuracy. In the case of the reciprocating engine (the Wright Aeronautical Corporation R1300-3) it was deemed advisable to follow practices developed in simulating dynamics of gas turbine engines, especially because of the dearth of material available on which to base a more detailed dynamical study. The purpose of this report is to indicate the lines along which the attempts to obtain a realistic transfer function progressed and the simulation which resulted from the efforts.

The present investigation was initiated with a study of the R1300-3 steady-state performance data which indicated that engine torque is a function principally of throttle angle and rpm. From this and the assumption of system linearity a simple transfer function involving no dynamics was derived.

A perusal of available treatments on the subject of engine dynamics resulted in a later decision to include dynamical terms in the functional relationship between throttle position and engine torque. Although this transfer function was used at the outset (reference 1) to simulate the helicopter engine, it was considered to be only a first approximation and one of problematical accuracy. Thus, it was subsequently proposed (reference 2) that dynamical tests be run on an available engine at the Navy's Aeronautical Engine Laboratory in Philadelphia. As a consequence, tests were conducted on a WAC R1820-74 and the data obtained were used as the basis for the derivation of the engine transfer function, as described in succeeding Sections of this report. Included here are discussions of testing procedure, resultant data, means for obtaining transfer function from data, deficiencies in present form, and suggestions for future work.

SECTION II

R1300-3 ENGINE PERFORMANCE CHARACTERISTICS

Steady-state performance data for the WAC R1300-3 engine are presented in Figures 1 and 2. Figure 1 is the standard form representation of the performance of the engine, relating brake horsepower, rpm, manifold pressure, and altitude. Figure 2 shows the relationship between engine torque, rpm, and throttle angle.

The first of the two equations which are basic to the present study is obtained from physical considerations and the above described performance data. It may be expressed as

$$Q_e = Q_e\,(N, Th) \tag{1}$$

where Q_e = engine torque, ft lbs.
N = angular velocity, rpm.
Th = throttle angle, degrees.

Also see Appendix A for further symbol definitions. The equation (1) expression for engine torque is required for solution of the second equation which defines the motion of the rotor and engine, and is written as follows:

$$Q_e - \frac{Q_A}{G} = \left(I_e + \frac{I_A}{G^2}\right) \frac{dN}{dt} \tag{2}$$

where Q_A = rotor aerodynamic or load torque, ft lbs.
I_e = engine polar moment of inertia, lb ft sec min rad/rev.
I_A = rotor polar moment of inertia, lb ft sec min rad/rev.
G = gear ratio, engine rpm to rotor rpm.
t = time, secs.

Consideration in the present analysis has been confined to the case of the engine operating at normal rated power at sea level; for the R1300-3, this corresponds to the following steady-state values:

Brake horsepower (BHP) = 700
Manifold pressure (P_M) = 39.5" Hg
Throttle angle (Th) = 38.5°
Angular velocity (N) = 2400 rpm

Although operation at only normal rated power has been considered here other operating points should be studied in the future to ensure satisfactory controlled responses since steady-state gains (such as the change in engine torque with throttle at constant rpm) can vary markedly with the selected operating point.

SECTION III

DEVELOPMENT OF ENGINE TRANSFER FUNCTION

3.1 Initial Assumption of No Dynamics

A very simple transfer function relating Q_e, Th, and N, which served to a large extent as the basis for the more complex functions although never actually used to simulate the engine, was developed through use of equation (1) and the assumption of engine linearity. Equation (1) was expanded as a total derivative resulting in:

$$Q_e = \left(\left.\frac{\partial Q_e}{\partial Th}\right|_{N\,=\,2400}\right) Th + \left(\left.\frac{\partial Q_e}{\partial N}\right|_{Th\,=\,38.5°}\right) N \qquad (3)$$

where all variables in equation (3) and henceforth represent changes from steady state conditions.

The coefficients

$$\left.\frac{\partial Q_e}{\partial Th}\right|_{N\,=\,2400} \quad \text{and} \quad \left.\frac{\partial Q_e}{\partial N}\right|_{Th\,=\,38.5°}$$

were determined from Figure 2 through graphic slope measurements at the operating point. Incidentally, an indication of the non-linearity which would be encountered in studying the engine control problem over the whole operating range may be obtained from an examination of the magnitude of these derivatives at the 80% and 90% as well as the 100% Normal Rated Power operating points (following the maximum engine economy curve):

TABLE I

Steady-State Engine Gains

NRP	BHP	N	Th	Q_e	$\left.\frac{\partial Q_e}{\partial Th}\right\|_N$	$\left.\frac{\partial Q_e}{\partial N}\right\|_{Th}$
100%	700	2400	38.5	1530	26.6	-0.15
90%	630	2315	34.0	1420	31.7	-0.25
80%	560	2225	30.0	1320	35.5	-0.35

This evidence of non-linearity, due to variation in engine gains with operating point, is also found during large amplitude transients which begin at an arbitrary operating point in that the gains vary with deviation from the set point. This will be further discussed in connection with the engine testing program later in this report.

The resultant transfer function at 100% NRP based on the assumption of no dynamics existing in the equation (3) relationship can be seen from Table I to be:

$$Q_e = 26.6\ Th - 0.15N \qquad (4)$$

As was indicated above, this function was never used in any of the controls analyses.

3.2 Consideration of Engine Dynamics

Considerable effort was made to establish whether it is necessary to include dynamics in the relationships $Q_e = f_1(Th)$ and $Q_e = f_2(N)$. Literature on the subject was carefully surveyed, including material available at ASTIA and at the Technical Data Branch of the Bureau of Aeronautics. Although aircraft reciprocating-engine acceleration data did not appear to be in existence, some acceleration data on automobile engines were found (see reference 3) which definitely indicated the need for a consideration of engine dynamics, at least in the relationship between Th and Q_e. Several text books (reference 4 and 5) acknowledge the existence of engine dynamics, in their discussion of carburetor accelerating wells. The purpose of these wells is to supply additional fuel, over and above normal requirements, in an attempt to overcome the lag in fuel vaporization immediately after a sudden throttle advance. Another text (reference 6) assumes dynamics between torque and throttle in a study of engine control but gives no concrete basis for the assumption. Additional bibliographical material is presented in Reference 1.

With the background obtained from the literature it was concluded that a consideration of dynamics is required for a study of engine control, at least in the throttle to torque portion of equation (3). This conclusion was supported by the findings in Reference 7, which also supplied a basis for actual assumption of a transfer function. Reference 7 contains a discussion of the simulation for control purposes of the engines on the B-36 and B-50 airplanes.

For the relationship between Th and Q_e, the steady-state gain transfer function as given in equation (3) was modified, following Reference 7, to include two first order lags, the time constants of which were to represent (1) the time required to cause a change in fuel-air mixture, and (2) the time required for the change in fuel-air flow to enter the engine, be ignited, and expelled. It was felt that, as a first approximation, these lags chosen in Reference 7 for a turbo-supercharged engine, could be reasonably applied to the R1300-3 engine, which has a single-stage single-speed supercharger. Engine acceleration tests later showed this assumption to be valid.

Literature could not be found which discussed the dynamical relationship between N and Q_e. There is reason to assume, however, that the dynamical effects of variation in rpm on engine torque are

small from the following considerations. To begin with, let it be presumed that the control criterion provides for a maximum allowable deviation of rpm during transients of 2% of the steady-state value. Two per cent of the steady-state rpm (2400) equals 48 rpm. This change in rpm gives a 0.15 x 48 = 7.2 ft. lb. change in torque. The effect, in turn, of a 7.2 ft. lb. change in torque on the engine rpm can be determined from the torque equation (2). As indicated in Appendix B, the total moment of inertia in terms of engine rpm (for the HRS installation) is 239.8 lb ft sec min rad/rev. The relationship

$$Q_e = I_{Tot} \frac{dN}{dt}$$

gives

$$\frac{dN}{dt} = \frac{Q_e}{I_{Tot}} = \frac{7.2}{239.8} = 0.03 \text{ rpm/sec.}$$

which corresponds to 0.03/11.3 = 0.003 rotor rpm per sec. Thus, if a 2% change in rpm took place (say, as a result of a gust load), the change in engine torque, even ignoring any time lags, would be relatively small (compared with the 2% initiating rpm change) in terms of its effect in turn on the rpm of the rotor system. It is felt that this (coupled with the lack of available data on this question) justified the omission of dynamics in the relationship between engine torque and rpm.

Actually, the steady-state term

$$\left.\frac{\partial Q_e}{\partial N}\right|_{Th = 48.5°}$$

was retained since it was felt that the engine acceleration tests might later indicate a need for dynamical terms and provision was thus made for their future insertion.

The transfer function derived from the above consideration was

$$Q_e = \frac{26.6}{(0.5S + 1)(0.044S + 1)} Th - 0.15N \qquad (5)$$

This function was used (see reference 1 - page 11) until another was developed from engine acceleration test data.

3.3 Transfer Function Obtained from Engine Data

Although the above engine transfer function had some logical basis of derivation, it was felt that experimental corroboration was required if the study of rotor rpm control was to be viewed with confidence. For this reason it was requested of the Bureau of Aeronautics and the Naval Aeronautical Engine Laboratory that dynamical

tests be conducted on the R1300-3 engine or an engine which has similar transient characteristics. It was proposed that two types of tests be run, (1) inputs to throttle, recording engine torque and throttle position, and (2) inputs to load torque, causing variations in engine rpm, while recording engine torque and engine rpm. These were further sub-divided into acceleration and deceleration runs.

3.3.1 Test Setup

The engine used in the AEL tests was the WAC R1820-74 which differs from the R1300-3 (the actual HRS-3 power plant) slightly in moment of inertia, since the former has two more cylinders than the latter, and greatly in torque output; the R1820-74 has an output under sea level take-off conditions of approximately 2700 ft. lbs., compared with 1620 ft lbs. for the R1300-3. Because the helicopter rotor (or load) moment of inertia is effectively much greater than the engine moment of inertia, it is reasonable to assume that small differences of moment of inertia in the two engines will have little meaning in the ultimate use of the dynamical data.

An electric dynamometer was chosen to simulate the dynamical characteristics of the rotor in preference to a propeller because the available propeller governor could not hold rpm to required tolerances, and the use of a propeller stand would have delayed tests by approximately one month.

The test setup was instrumented to the extent that oscillograph traces of load torque, rpm, and throttle motion could be obtained. Load torque was measured by strain gages as the reaction of the dynamometer housing. The position of the carburetor butterfly valve or throttle was measured by a potentiometer. RPM was obtained through use of an AC tachometer, the output of which was rectified, the complete network having a single order lag of 0.179 sec. time constant. The tachometer lag was considered to be the only dynamics involved in the instrumentation. Further discussion on the subject of instrumentation will no doubt be included in the AEL final test report.

3.3.2 Test Procedures

In considering various test procedures, AEL did not favor either sinusoidal or step inputs for this work. Sinusoidal inputs were considered too difficult to obtain with the available setup and the complication involved in trying to reinstrument the test stand would have required an inordinate amount of time, money, and effort. Step inputs were considered to be of such a nature that good results would not be obtainable with their use. (Later in the test program so-called snap inputs in throttle were used for acceleration runs). Agreement was reached in the use of ramp inputs for both throttle and load.

The variation in engine torque with throttle was obtained by manual ramp inputs to throttle through a linear linkage. Stops on the throttle lever were set at the base operating point and at another operating point corresponding to several degrees change in throttle.

Because the derivative had to be determined at constant rpm, it was necessary for the operator to become adept at varying the load torque with one hand through variations in the dynamometer field while moving the throttle lever smoothly between stops with the other. Since both rpm and throttle position were measured, an accurate account of the operator's success was available. Four good acceleration and four good deceleration runs were chosen for each operating point from the data which were obtained. The engine torque was derived from a torque meter. It can be readily seen that with the above operation, sinusoidal or step inputs would have been very difficult for an operator to follow. However, to obtain a smooth ramp it was necessary to move the throttle several degrees, which apparently took the engine out of the linear range. Thus, in the final stages of the test program, AEL did switch to the small step-type input. It is assumed that the operator held rpm constant in this case much in the same manner as before.

The change in engine torque with change in rpm was found by varying dynamometer field strength manually to simulate a ramp change in load. The load change caused a change in recorded rpm which in turn was presumed to affect the engine torque. Throttle position was maintained constant. The shape of the rpm 'input' was not a simple mathematical function; this consequently caused some complication in the analysis, as discussed later. As before, four runs were made for acceleration and deceleration at each operating point considered (100%, 90% and 70% normal rated power).

The following data were transmitted to Minneapolis-Honeywell as a result of these AEL tests.

TABLE II

AEL Engine Data Received by Minneapolis-Honeywell

No. of Runs		Type of Input	Oper. Pt.	Date Received
Accel.	Decel.			
1		Th	100% NRP	24 Nov. 1953
1		Qa	100% NRP	24 Nov. 1953
1	1	Th	90% NRP	8 Jan. 1954
1	1	Qa	90% NRP	8 Jan. 1954
4		Th	90% NRP	23 Feb. 1954
4		Th	70% NRP	23 Feb. 1954

3.4 Engine Data Analysis

The simulation of reciprocating engine dynamics requires four types of data:

TABLE IIIa

Types of Engine Data Received

Type No.	Type of test run
(1)	response of Q_e to increase in Th
(2)	response of Q_e to decrease in Th
(3)	response of Q_e to increase in N
(4)	response of Q_e to decrease in N

For analysis purposes the data thus far received from AEL may be listed more conveniently in the following form:

TABLE IIIb

Set	Type No	No. of Runs	Input	Size of Input	Oper. Pt.	Date Received
a.	1	1	Ramp	Large	100% NRP	24 Nov. 1953
b.	1	1	Ramp	Large	90% NRP	8 Jan. 1954
c.	1	4	Step	Small	90% NRP	23 Feb. 1954
d.	1	4	Step	Small	70% NRP	23 Feb. 1954
e.	2	1	Ramp	Large	90% NRP	8 Jan. 1954
f.	3	1	Ramp	Large	100% NRP	24 Nov. 1953
g.	3	1	Ramp	Large	90% NRP	8 Jan. 1954
h.	4	1	Ramp	Large	90% NRP	8 Jan. 1954

It is immediately clear that only small amounts of deceleration data have been received. The size of input is included as one of the headings in Table IIIb since it is well to note that the engine was well into its non-linear range for those marked "large". Other points of interest concerning the data will be mentioned in the course of the following analysis.

The general analysis procedure was aimed at finding a simulation on the REAC which produced a trace which closely resembled the variation in engine torque for throttle or rpm inputs similar to those actually experienced by the engine. The results of the REAC work were then used to determine a transfer function.

$\left.\frac{\partial Q_e}{\partial Th}\right|_N$ (acceleration): This function was determined from data type 1 for which there are four sets of data runs. Data set c. (see Table IIIb) was chosen since a small step was used (thereby preserving a linear engine torque variation), and because the tests were run at the 90% operating point. In the over-all controls study under Nonr-929(00) it has been assumed that the R1300-3 engine is operating at 100% N.R.P.; since complete dynamical test results are available only for 90% N.R.P., the additional assumption is made that the differences in dynamics between the R1300-3 and the R1820-74 at the 100% N.R.P. point overshadow any differences in the R1820-74 dynamics between the 100% and 90% N.R.P. points.)

The REAC diagram which was used to simulate this portion of the engine is shown in Figure 3. Provisions are shown for both ramp and step inputs because data for both input types were available. In Figures 4 and 5, the engine inputs and responses and final simulations thereof are compared. Data set c. is shown in Figure 4, while in Figure 5 the transfer function obtained from set c. is subjected to a ramp input and compared with the set b. data. The set b. data were derived from a large throttle ramp* which took the engine into its non-linear range; thus, not too much dependence can be placed on this set of data. It is believed that the simulated dynamical characteristics of the engine as indicated in Figures 4 and 5 are sufficiently similar to that found in the experiments as to satisfy the present requirements.

The transfer function is obtained from the potentiometer settings shown on Figure 3 as

$$Q_e = \frac{26.6\ Th + 66.5\ \dot{Th}}{\frac{S^2}{3.6} + 1.06\ S + 1} \qquad (6)$$

where throttle is in degrees. The numerator gains were obtained by assuming steady-state

$$\left.\frac{\partial Q_e}{\partial Th}\right|_N = 26.6$$

(Table I) and then determining

$$\left.\frac{\partial Q_e}{\partial \dot{Th}}\right|_N$$

through comparison of the relative voltages for the two derivatives as obtained from settings of potentiometers (5) and (1), Figure 3.

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* Actual throttle amplitude in test was 3.4°; by presuming the torque to vary linearly, it was possible to reduce the data in Plate 1 of Reference (8) to that shown in Figure 5.

$\left.\frac{\partial Q_e}{\partial Th}\right|_N$ Decel.: Only one data set (set e.) has been received for this type test and that consists of only one run. This run was made at 90% N.R.P., but a large ramp input (5 1/2° Th) was used; consequently, the engine transient went well into the non-linear range. Little effort was expended in trying to duplicate set e. on the REAC because of the non-linearities involved and because data of the small step-input variety were believed to be available at AEL. These data, of course, will be much more useful to us. The REAC diagram shown in Figure 6 gives a response to a ramp input which follows closely that of the engine for the first two seconds, during which the engine torque change is still in the linear range. However, it is not useful in obtaining the transfer function because steady-state conditions were not reached by the simulated response. The results are shown along with engine data on Figure 7. Further work on this function must await receipt of the appropriate AEL data.

$\left.\frac{\partial Q_e}{\partial N}\right|_{Th}$ (Accel. and Decel.): Types 3 and 4 test runs are discussed together because of the many similarities involved. In both cases only a minimum of engine data have been received. The runs which are available show responses to large inputs which run the engine into the non-linear regime. Another failing in these runs, which may not be readily eliminated, is related to the fact that as rpm varies, the load torque varies, which causes further variation in rpm and thus in the observed engine torque. For these reasons only the first few seconds of each run are used for simulation purposes, the rpm having relatively low values during these periods.

A REAC diagram and transfer function which can be used for both types 3 and 4 are shown in Figure 8, along with the appropriate pot settings. Comparison of REAC results and engine traces for type 3 data are shown in Figure 9. Figure 10 shows the same information for type 4 data. As is readily seen in the figures, the test results were modified after the first few seconds and the runs were extended somewhat arbitrarily to give a steady-state value upon which the transfer function calculations could be based. Figure 11 shows a run which approximates an average between types 3 and 4. The simulation for Figure 11 might be used to eliminate the non-linearity with acceleration.

The transfer function for types 3 and 4 is of the form

$$Q_e = \frac{K_1 + K_2 S + K_3 S^2}{(\tau_3 S + 1)} N \qquad (7)$$

where N is in engine rpm. For the various runs involved, assuming K_1 again equals -0.15 (Table I), the gains and time constant become:

	K_1	K_2	K_3	τ_3
Accel.	-0.15	-0.045	-0.75	0.2
Decel.	-0.15	-0.05	-0.0	0.2
Ave.	-0.15	-0.05	-0.35	0.2

SECTION IV

CONCLUSIONS AND RECOMMENDATIONS

The transfer function obtained from the results of the type 1 data and an average of the types 3 and 4 data was as follows:

$$Q_e = \frac{26.6 + 66.5\,S}{\frac{S^2}{3.6} + 1.06S + 1}\,Th - \frac{0.15 + 0.05S + 0.35S^2}{(0.2S + 1)}\,N \qquad (8)$$

The function presently being used in the helicopter controls studies under Ncnr-929(00) is similar to that of equation (8) except that the second term is simply -0.15N.

The relationship between engine torque and throttle for acceleration is the only one of the four types studied in which there is a substantial degree of confidence for reasons given in the previous Section. Additional data, which are expected from AEL, would make possible the inclusion of a deceleration function for Q_e (= Q_e (Th)) in a satisfactory manner.

Concerning the relationship between torque and rpm, it appears that further data from the test procedure employed in the present experiments will not necessarily lead to an improved function. Some means should be found to eliminate the effect on engine torque of the variation in load torque with rpm. In any event, it appears that the dynamics in the relationship between engine torque and rpm may be negligible.

The complete transfer function should be determined for another operating point, say 70% of normal rated power, especially for application to automatic controls studies for multi-engine helicopters.

SECTION V

REFERENCES

1. Warsett, P., Brown, C., and Albachten, H., "Study of Automatic Control Systems for Helicopters--Final Report on Phase I", M-H Aero Report AD 5143-TR8, 31 July 1953.

2. Brown, C. A. (Minneapolis-Honeywell letter) to Aeronautical Engine Laboratory (Attn: Messers Sanwald and Taylor), re: Engine tests requested to be conducted at AEL; 28 May 1953.

3. Eisinger, J. O., "Engine Acceleration Tests", Trans SAE Vol. 22 Part 2, 1927.

4. Taylor, C. F. and Taylor, E. S., "The Internal Combustion Engine", International Textbook Co., Scranton, 1948.

5. Judge, A. W., "Automobile and Aircraft Engines", Sir Isaac Pitman and Sons, Ltd., 1937.

6. Ahrendt and Laplin, "Automatic Feedback Control", McGraw-Hill Book Company, Inc., New York, 1951.

7. Clemens, J. E. et al, "Automatic Control of Aircraft Engines with Turbo Supercharger," AMC, OAR Technical Report, No. 2, 1 November 1950.

8. Sager, J. P., "Prelim. Report, Project TED No. NAM PP3508." Naval Air Material Center Letter Report XE-1-HDR:ap, F11-1/2/1820, F22. 23 December 1953.

APPENDIX A

SYMBOLS

G ----- gear ratio

I_e ----- engine moment of inertia

I_a ----- load (rotor) moment of inertia

K_1 ----- proportional gain

K_2 ----- rate gain

K_3 ----- acceleration gain

N ----- angular velocity

P_m ----- manifold pressure

Q_e ----- engine torque

Q_a ----- load torque

T_h ----- throttle angle

s ----- Lapacian operator

t ----- time

τ ----- time constant

APPENDIX B

CALCULATION OF MOMENT OF INERTIA FOR HRS ENGINE LOAD

A. Main Rotor and Tail Rotor

	Gear Reduction	Moment of Inertia
Main Rotor	11.315	2775 slug ft^2
Tail Rotor	1.62	2.5 slug ft^2

taken relative to main rotor and tail rotor angular velocity, respectively. Converting the tail rotor moment of inertia to be relative to main rotor angular velocity, this becomes

$$\left(\frac{11.315}{1.62}\right)^2 \times 2.5 = 122.1 \text{ slug ft}^2$$

B. Engine (moment of inertia relative to engine rad/sec)

Basic engine:	2200	lb. in^2
Supercharger:	2120	lb. in^2
Flywheel :	7140	lb. in^2
Total	11460	lb. in^2

Converting to main rotor angular velocity (rad/sec), the engine moment of inertia becomes

$$(11.315)^2 \times 11460 \times \frac{1}{32.2 \times 144} = 316.5 \text{ slug ft.}^2$$

C. Total Moment of Inertia

$$316.5 + 2775 + 122.1 = 3213.6 \text{ slug ft}^2 \text{ (relative to rotor rad/sec)}$$

Converting this total in terms of engine rpm gives

$$\frac{3213.6}{(11.315)^2} \times \frac{60}{2\pi} = 239.8 \text{ lb ft sec min rad/rev.}$$

Brake Horsepower vs Manifold Pressure at Constant RPM for R1300-3 Engine

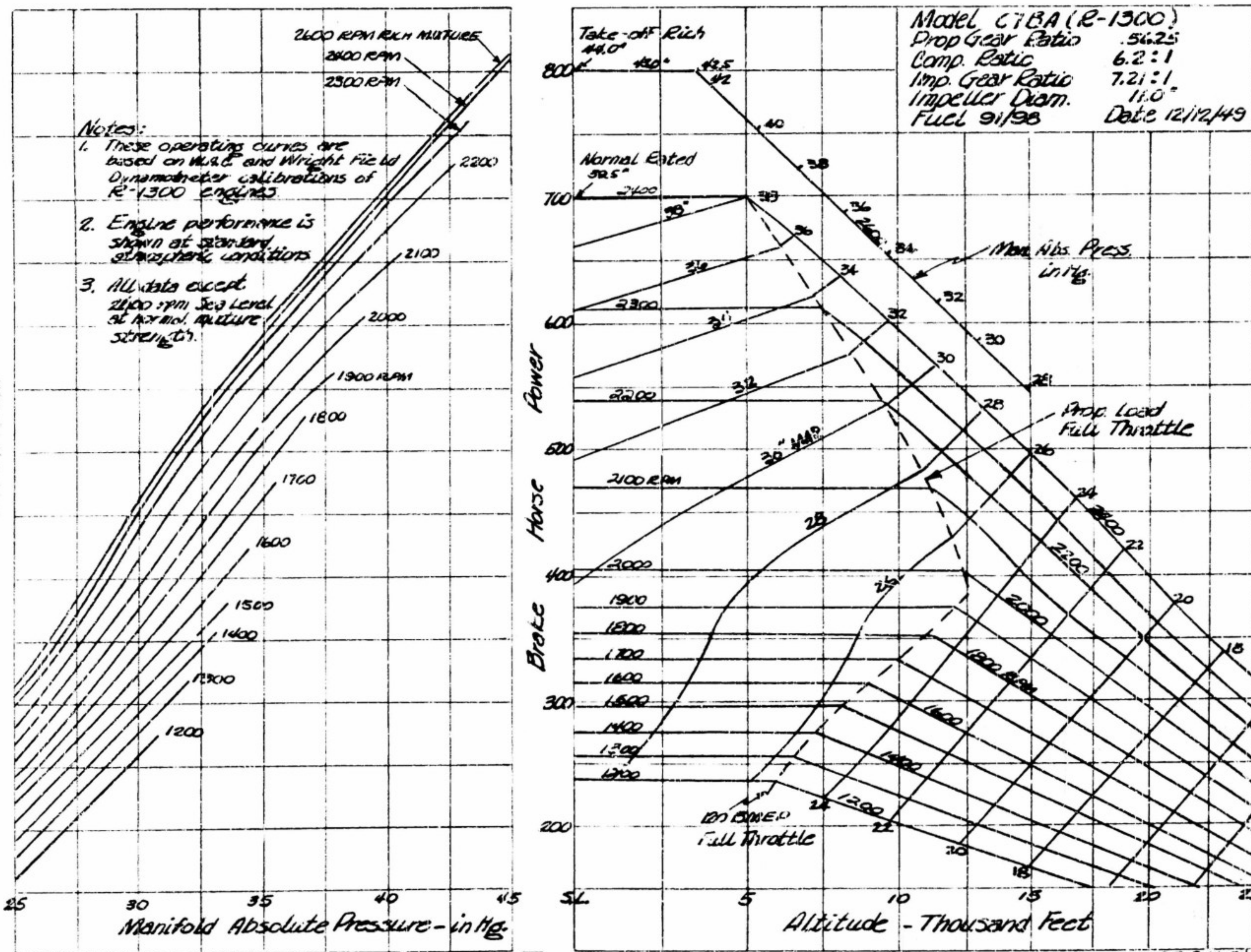
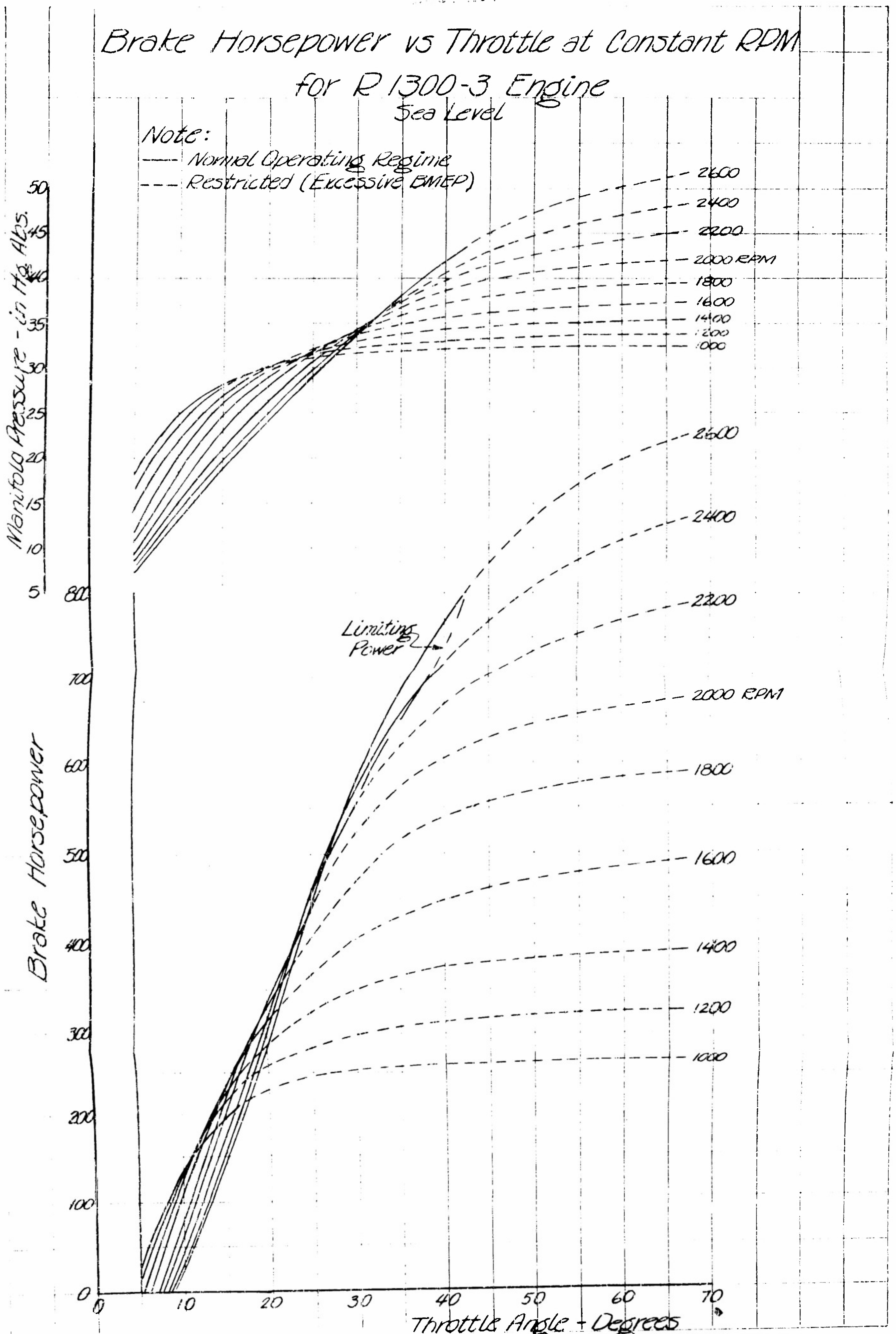


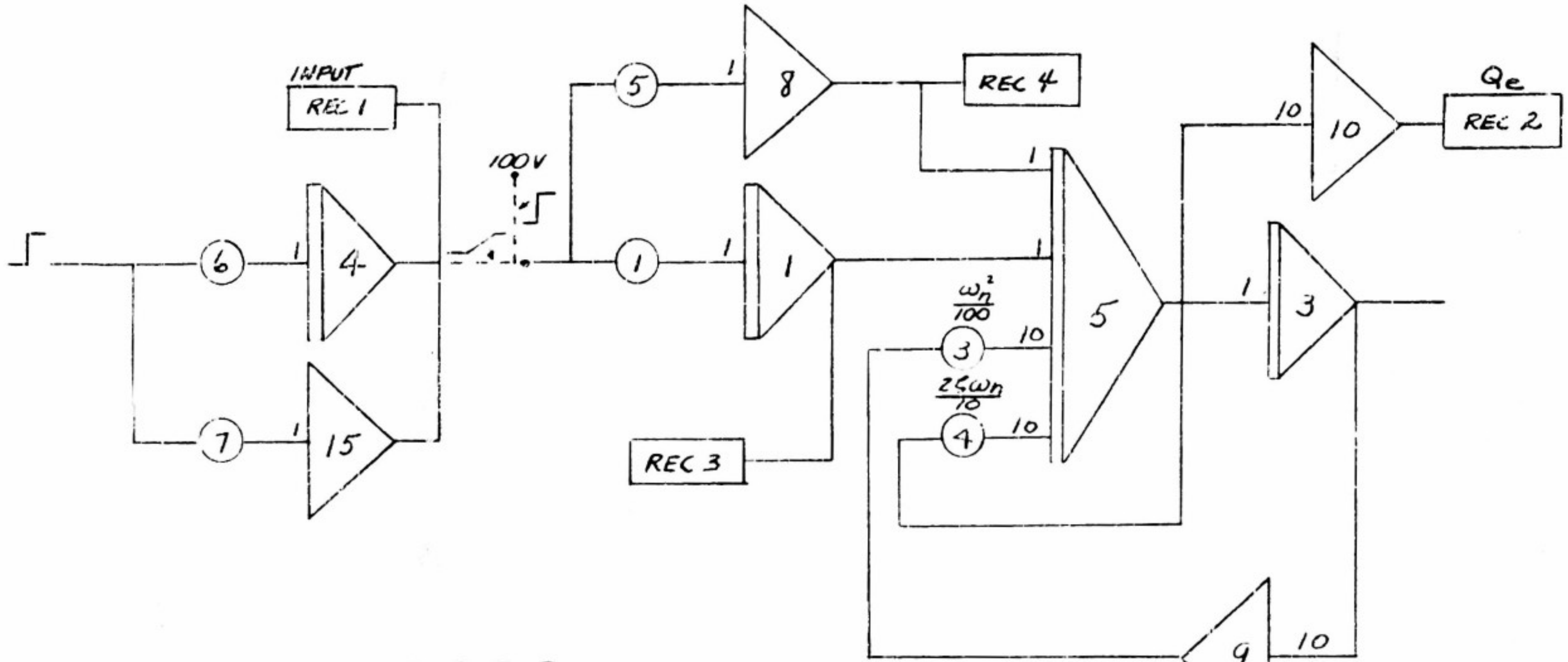
FIGURE 1

CONFIDENTIAL



Brake Horsepower vs Throttle at Constant RPM
for R 1300-3 Engine
Sea Level
Note:
— Normal Operating Regime
- - - Restricted (Excessive BMEP)
Manifold Pressure - in Hg Abs.
Brake Horsepower
Throttle Angle - Degrees
Limiting Power
2600
2400
2200
2000 RPM
1800
1600
1400
1200
1000

FIGURE 2



FIGS.	1	3	4	5	6	7	INPUT
4	.06	.036	.380	.150	-	-	
5					.266	.5	

$$Q_e = \frac{K_1/\omega_n^2 + (K_2/\omega_n^2)S}{\frac{S^2}{\omega_n^2} + \frac{2\zeta\omega_n S}{\omega_n^2} + 1} Th$$

FIGURE 3: REAC Diagram for Type I Data (See Table III)

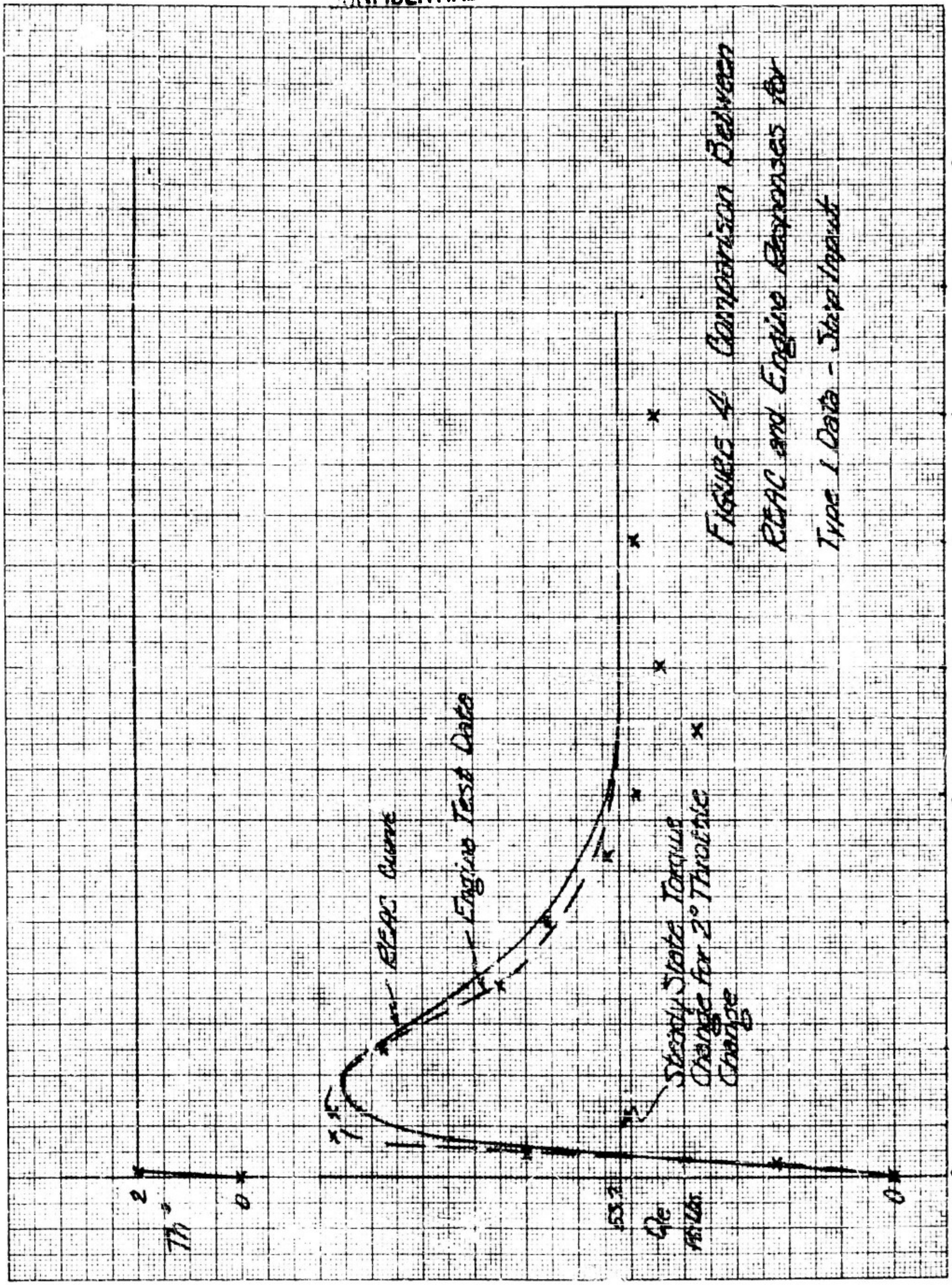


FIGURE 4 Comparison Between REAC and Engine Responses for Type I Data - Step Input

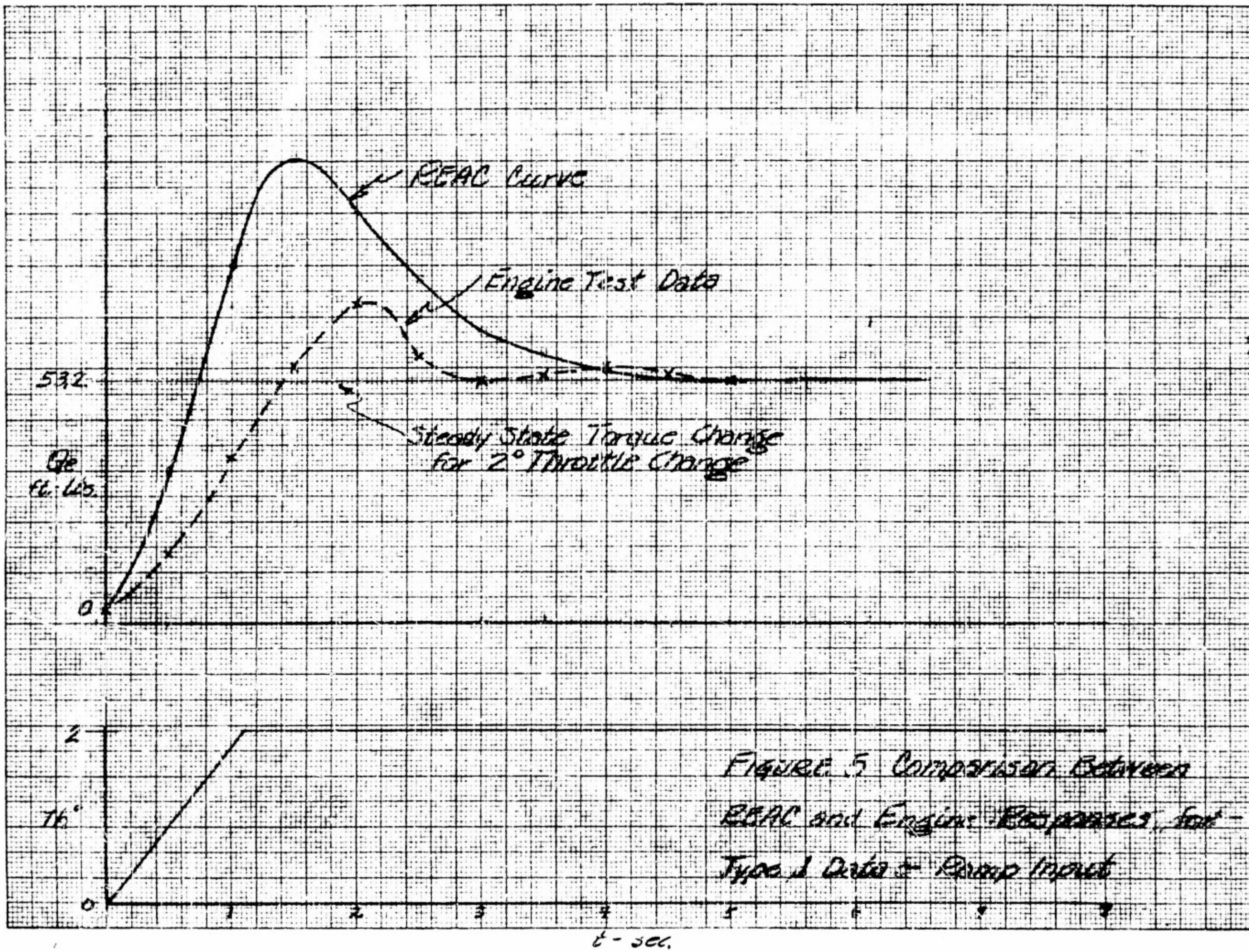


Figure 5 Comparison Between REAC and Engine Responses for Type I Data + Ramp Input

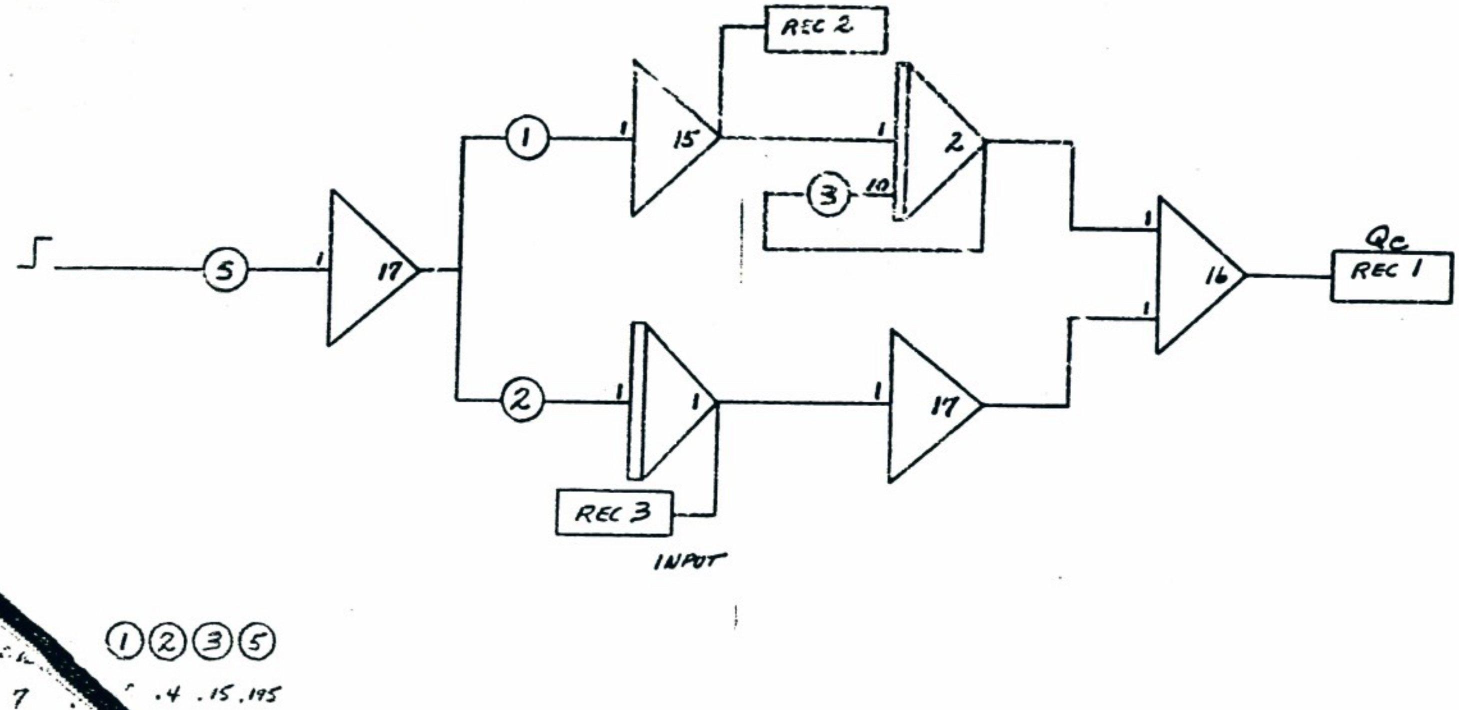


FIGURE 6 REAC Diagram for Type 2 Data

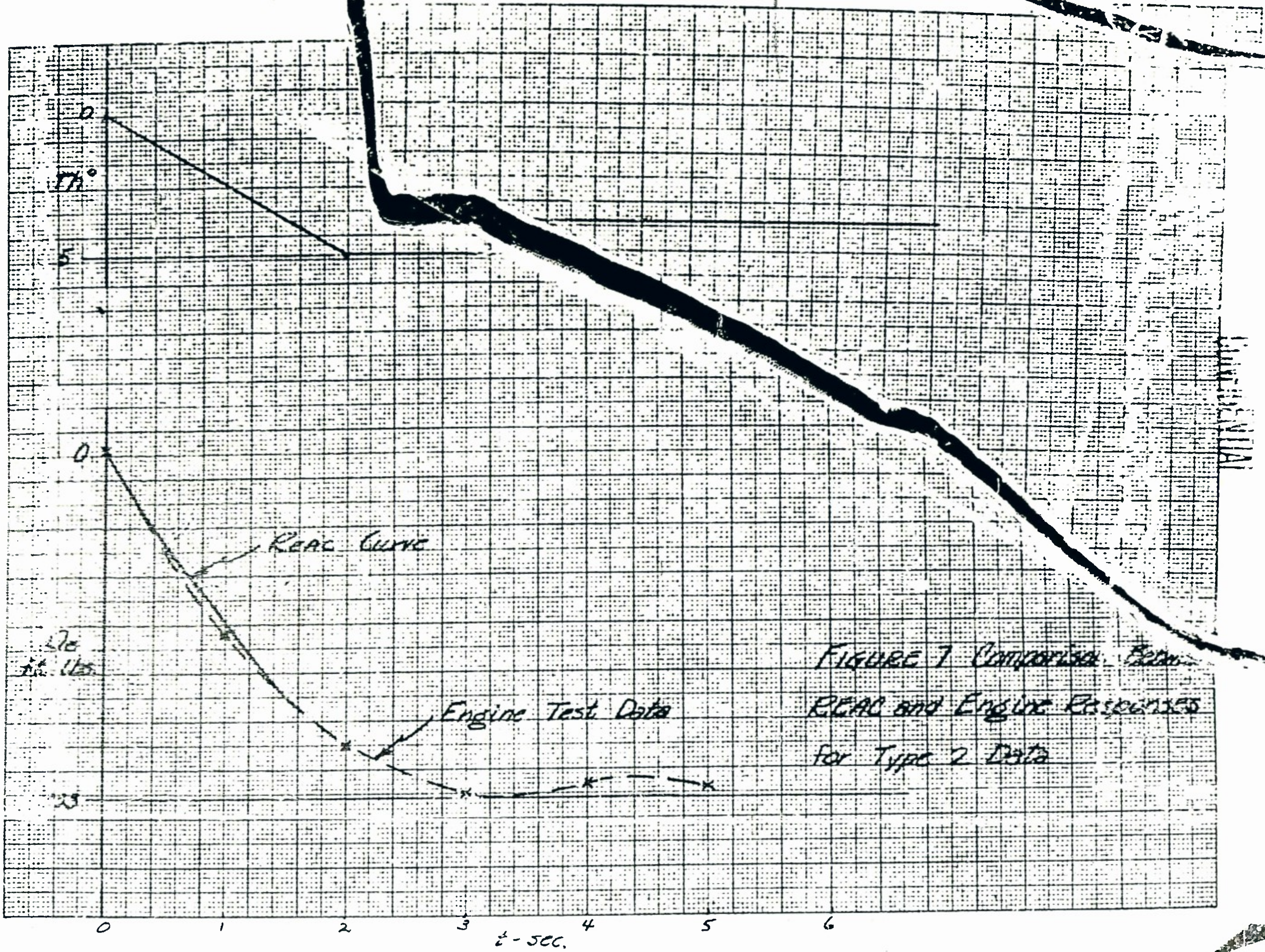


FIGURE 7 Comparison Betw
REAC and Engine Responses
for Type 2 Data

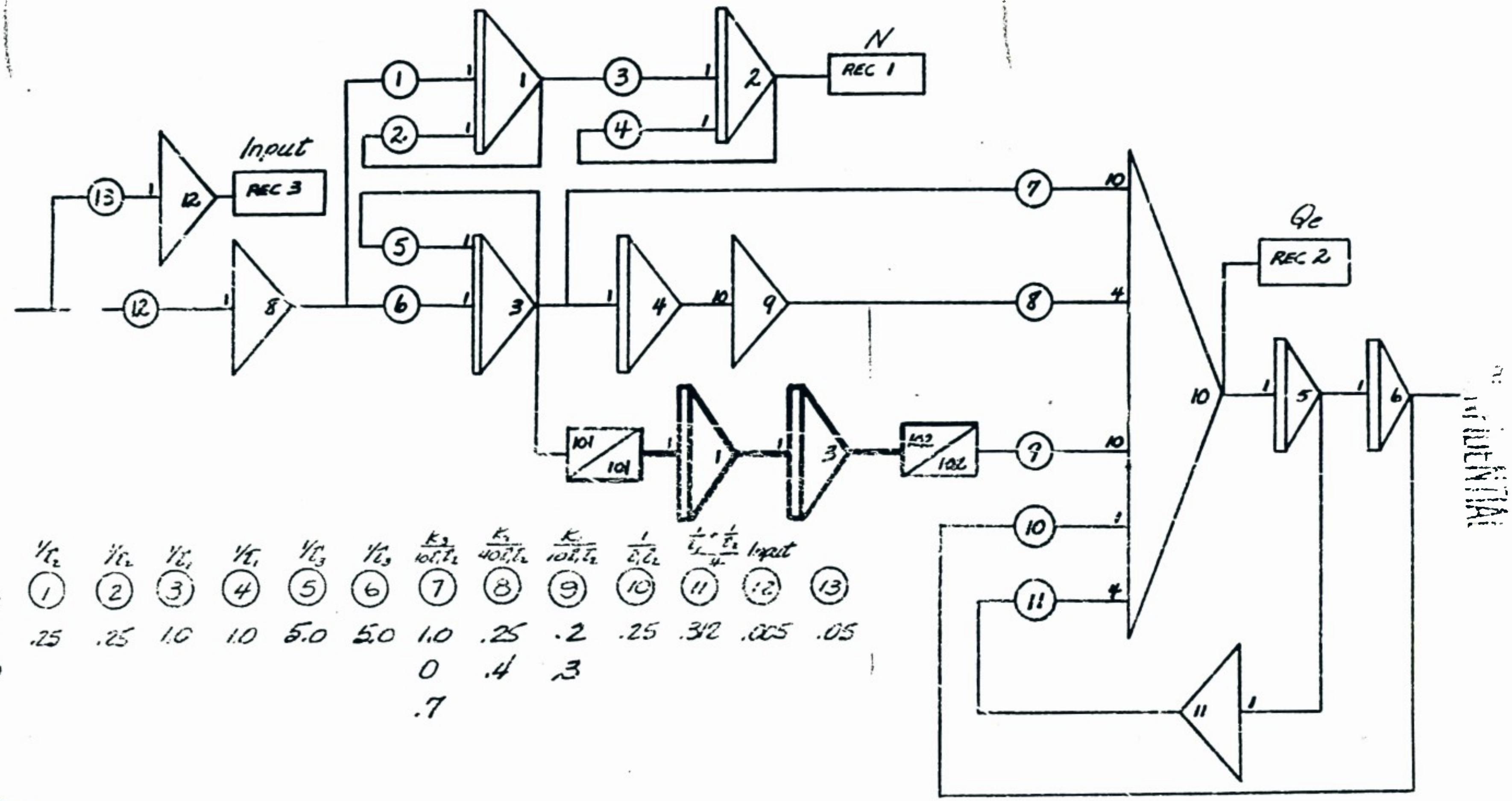


FIG.	① $1/\tau_2$	② $1/\tau_2$	③ $1/\tau_1$	④ $1/\tau_1$	⑤ $1/\tau_3$	⑥ $1/\tau_3$	⑦ $\frac{K_3}{10\tau_1\tau_2}$	⑧ $\frac{K_2}{40\tau_1\tau_2}$	⑨ $\frac{K_1}{10\tau_1\tau_2}$	⑩ $\frac{1}{\tau_1\tau_2}$	⑪ $\frac{\frac{1}{\tau_1}+\frac{1}{\tau_2}}{4}$	⑫ Input	⑬
9	.25	.25	1.0	1.0	5.0	5.0	1.0	.25	.2	.25	.312	.005	.05
10							0	.4	.3				
11							.7						

$$Q_e = \frac{K_1/\tau_3 + (K_2/\tau_3)S + (K_3/\tau_3)S^2}{(1/\tau_3 + S)} \cdot \frac{1/\tau_1\tau_2}{(1/\tau_1 + S)(1/\tau_2 + S)} \text{ (Input) for analysis}$$

$$\frac{Q_e}{S^2} = \frac{K_1/\tau_3 S^2 + K_2/\tau_3 S + K_3/\tau_3}{(1/\tau_3 + S)} \cdot \frac{1/\tau_1\tau_2}{(1/\tau_1 + S)(1/\tau_2 + S)} \text{ (Input) simulated}$$

FIGURE 8 REAC Diagram for Types 3 & 4 Data

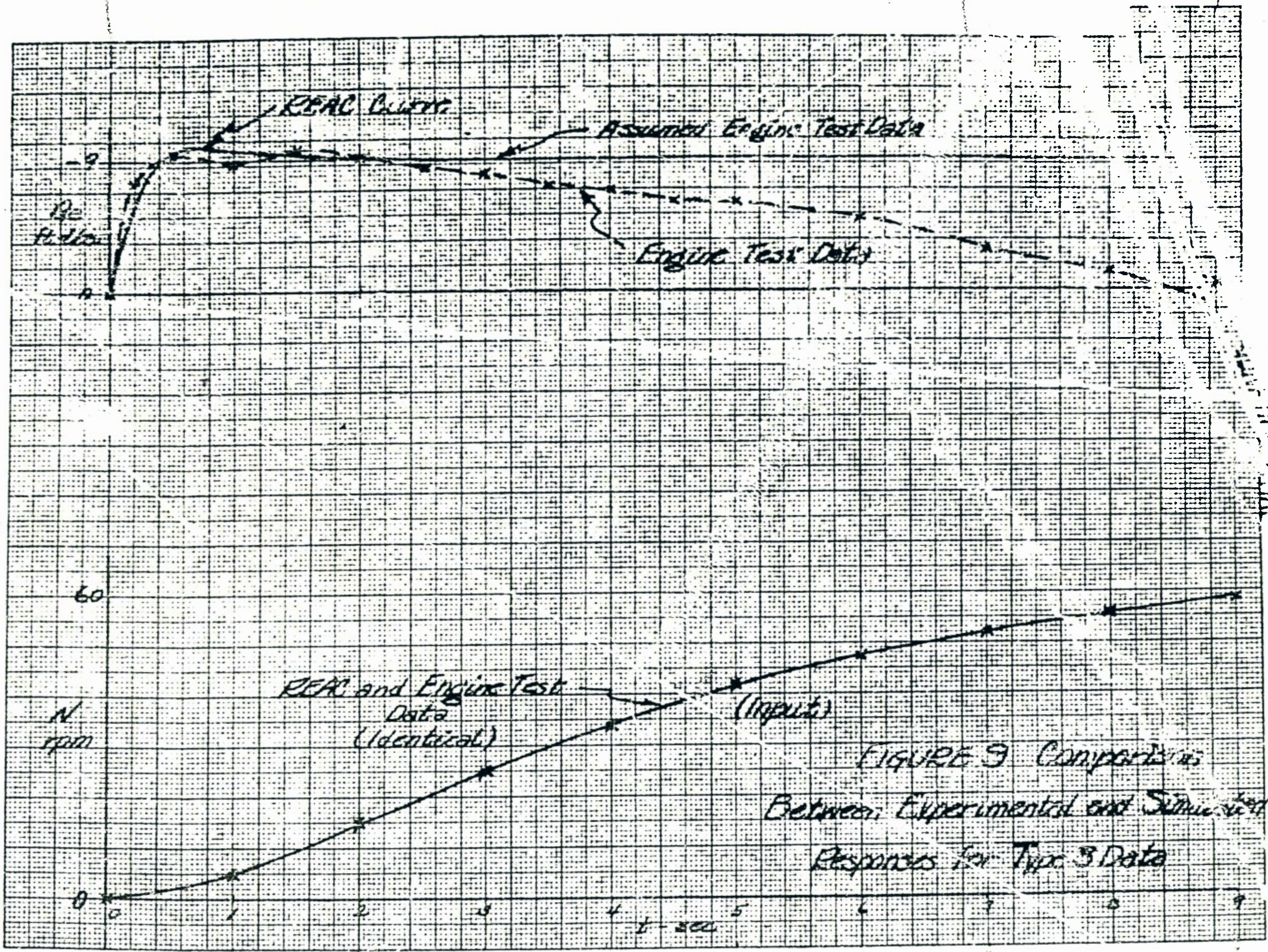


FIGURE 9 Comparison Between Experimental and Simulated Responses for Type 3 Data

CONFIDENTIAL

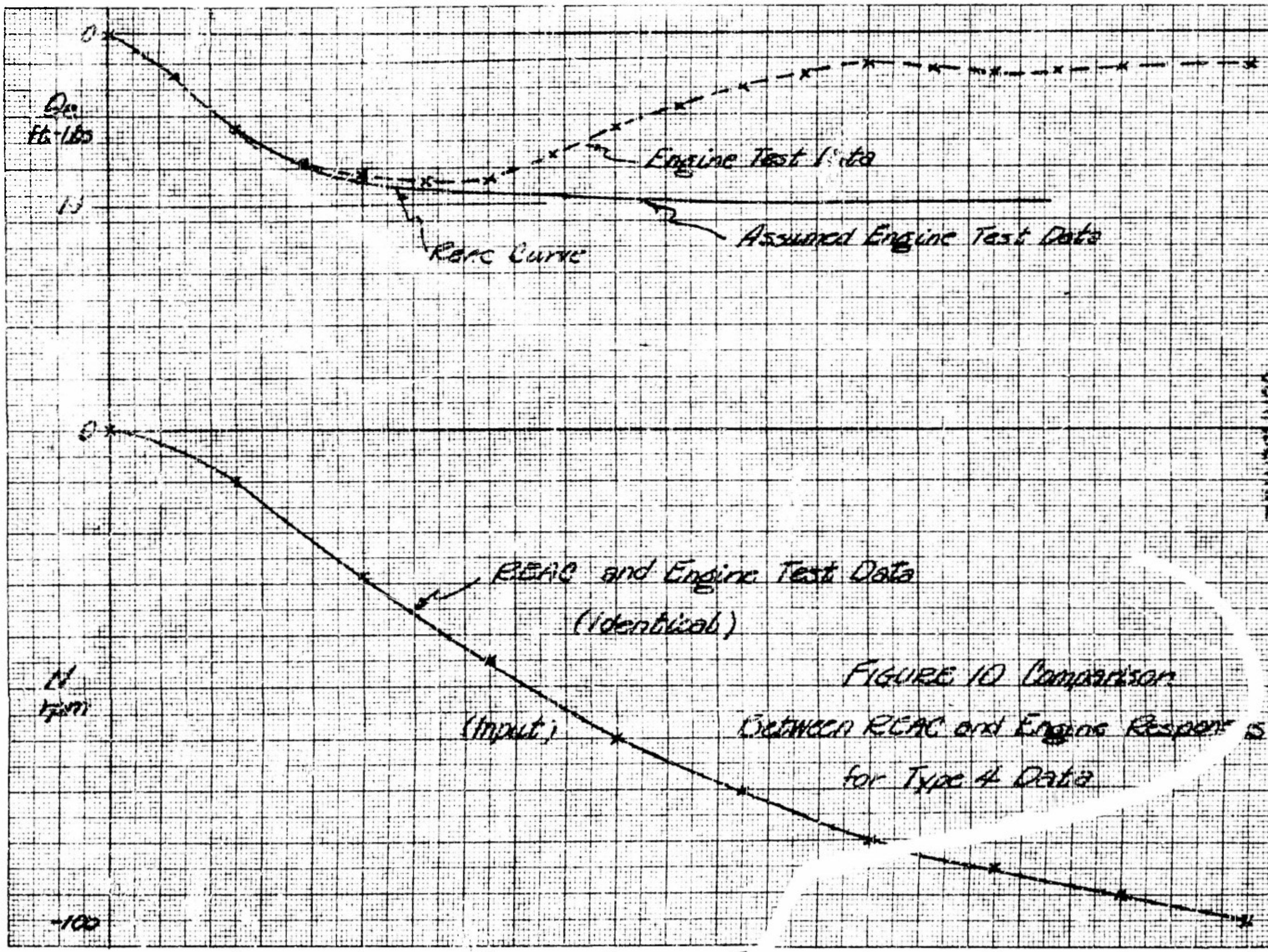


FIGURE 10 Comparison Between REAC and Engine Responses for Type 4 Data

CONFIDENTIAL

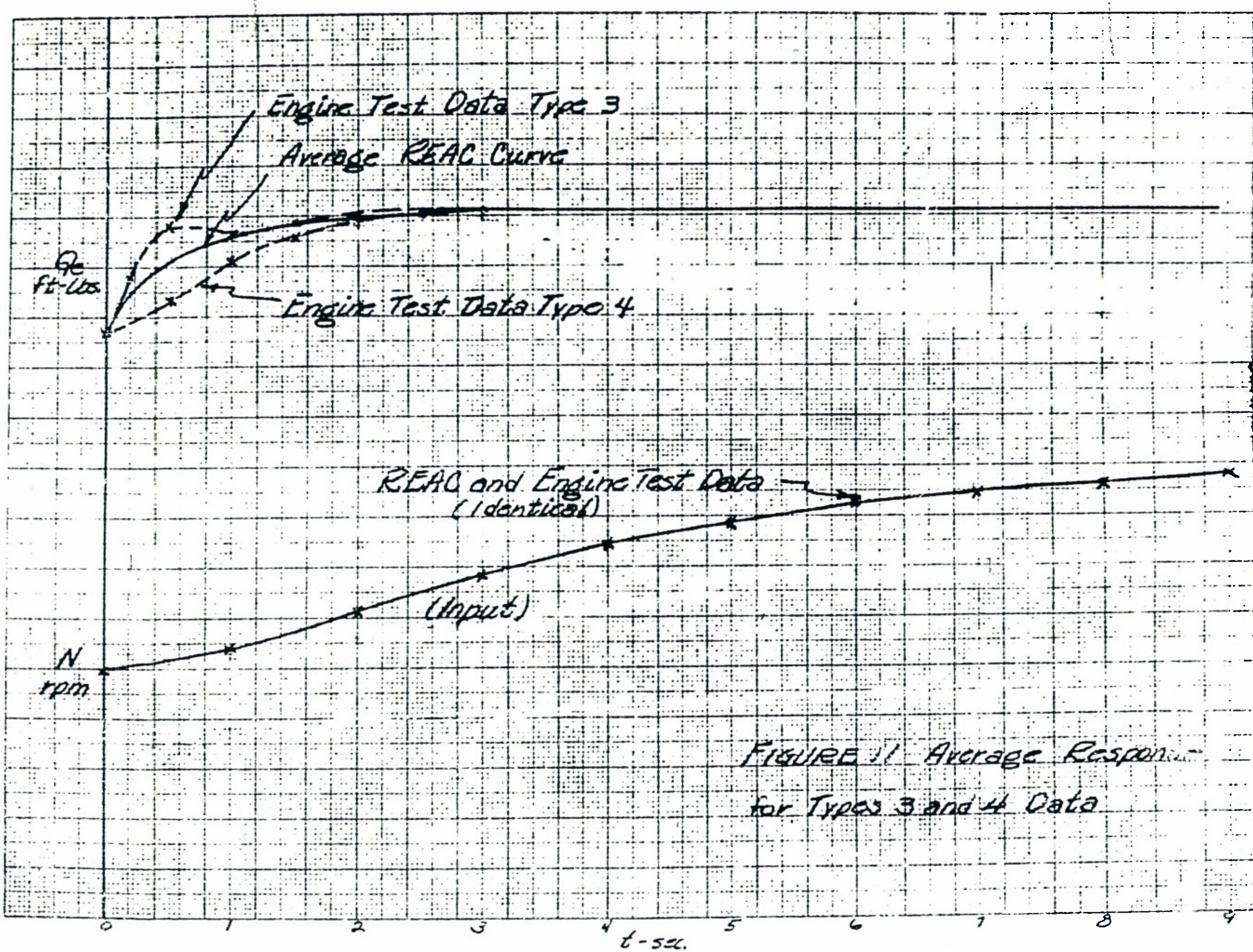


FIGURE 11 Average Response for Types 3 and 4 Data